

## On weak orbitally and asymptotically continuous mappings

BHARTI JOSHI<sup>✉</sup>0000-0002-6209-0107,  
NAVEEN CHANDRA<sup>\*✉</sup>0000-0003-0501-3401

ABSTRACT. In this note, we show that the recent results due to Khan and Oyetunbi [J. Fixed Point Theory Appl. 22:47, 2020], Górnicki [J. Fixed Point Theory Appl. 22:8, 2020], Bisht [J. Fixed Point Theory Appl. 21:54, 2019] and Górnicki [J. Fixed Point Theory Appl. 21:29, 2019] are still valid if mappings are not necessarily orbitally continuous or  $k$ -continuous on a complete metric space, but satisfy the weaker version of orbital continuity and  $k$ -continuity.

### 1. INTRODUCTION AND PRELIMINARIES

We recall some weaker forms of continuity as follows:

**Definition 1** ([2, 3]). If  $T$  is a self-mapping of a metric space  $(X, d)$ , then the set  $O(T, x) = \{T^n x : n = 0, 1, 2, \dots\}$  is called the orbit of  $T$  at  $x$  and  $T$  is called orbitally continuous at  $z \in X$  if for any sequence  $\{x_n\} \subset O(T, x)$  for some  $x \in X$ ,  $x_n \rightarrow z$  implies  $Tx_n \rightarrow Tz$  as  $n \rightarrow \infty$ , that is  $z = \lim_i T^{m_i} x$  implies  $Tz = \lim_i TT^{m_i} x$ .

**Definition 2** ([5]). Let  $(X, d)$  be a metric space and  $T : X \rightarrow X$ . A mapping  $f : X \rightarrow \mathbb{R}$  is said to be  $T$ -orbitally lower semi-continuous at a point  $z \in X$  if  $\{x_n\}$  is a sequence in  $O(T, x)$  for some  $x \in X$ ,  $\lim_{n \rightarrow \infty} x_n = z$  implies  $f(z) \leq \liminf_{n \rightarrow \infty} f(x_n)$ .

**Definition 3** ([9]). A self-mapping  $T$  of a metric space  $(X, d)$  is called  $k$ -continuous,  $k = 1, 2, 3, \dots$  if  $T^k x_n \rightarrow Tt$  whenever  $\{x_n\}$  is a sequence in  $X$  such that  $T^{k-1} x_n \rightarrow t$ .

---

2020 *Mathematics Subject Classification*. Primary: 47H10; Secondary: 54H25.

*Key words and phrases*. Orbitally continuous,  $T$ -orbitally lower semi-continuous, Weak orbitally continuous,  $k$ -continuous, Asymptotically continuous, Asymptotically regular.

*Full paper*. Received 23 Jan 2026, accepted 24 Apr 2026, available online 23 May 2026.

\*Corresponding Author.

**Remark 1.** A continuous mapping is orbitally continuous but not conversely (see [2,9]). Note that  $k$ -continuity of a mapping implies  $(k+1)$ -continuity for  $k \in \mathbb{N}$  while converse is not true (see [2]). Clearly, 1-continuity is equivalent to continuity.

**Remark 2** (Proposition 1 and Example 1 of [8]). Let  $(X, d)$  be a metric space,  $T : X \rightarrow X$  and  $z \in X$ . If  $T$  is orbitally continuous at  $z$  or  $T$  is  $k$ -continuous at  $z$  for some  $k \geq 1$ , then the function  $x \rightarrow f(x) := d(x, Tx)$  is  $T$ -orbitally lower semi-continuous at  $z$ . However, the converse may not be true. That is,  $T$ -orbital lower semi-continuity of  $x \rightarrow d(x, Tx)$  is weaker than both orbital continuity and  $k$ -continuity of  $T$ .

Moreover, in 2019, Pant et al. [9] introduced the following notion of weak orbitally continuous mappings.

**Definition 4** ([9]). A self-mapping  $T$  of a metric space  $(X, d)$  is called weak orbitally continuous if the set  $\{y \in X : \lim_i T^{m_i}y = u \Rightarrow \lim_i TT^{m_i}y = Tu\}$  is non-empty whenever the set  $\{x \in X : \lim_i T^{m_i}x = u\}$  is non-empty.

The following example shows that there exists  $T : X \rightarrow X$  such that  $T$  is weak orbitally continuous but  $T$  is neither orbitally continuous nor  $k$ -continuous.

**Example 1** ([9]). Let  $X = [0, 2]$  equipped with the Euclidean metric. Define  $T : X \rightarrow X$  by

$$T(x) = \frac{1+x}{2} \text{ if } x < 1, \quad T(x) = 0 \text{ if } 1 \leq x < 2, \quad T(2) = 2.$$

Here we observe that  $T^n 0 \rightarrow 1$  and  $T(T^n 0) \rightarrow 1 \neq T1$ . Therefore  $T$  is not orbitally continuous. However  $T$  is weak orbitally continuous. If we take  $x = 2$  then  $T^n 2 \rightarrow 2$  and  $T(T^n 2) \rightarrow 2 = T2$  and, hence  $T$  is weak orbitally continuous. Further,  $T$  is also not  $k$ -continuous. To see this, consider the sequence  $\{T^n 0\}$  then for any integer  $k \geq 1$ , we have  $T^{k-1}(T^n 0) \rightarrow 1$  and  $T^k(T^n 0) \rightarrow 1 \neq T1$ .

Recently, Pant et al. [10] also introduced an another weaker version of continuity named as the asymptotic continuity, which is the following.

**Definition 5** ([10]). A function  $T : X \rightarrow Y$  is said to be asymptotically continuous (or equivalently, asymptotically  $k$ -continuous) if  $\lim_{k,n \rightarrow \infty} T(T^k x_n) = Tu$  whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{k,n \rightarrow \infty} T^k x_n = u$ .

However, we note that continuity  $\implies$   $k$ -continuity  $\implies$  asymptotic continuity but not conversely, for the sake of convenience one can see the Example 1.8 given in [10]. Additionally, it is also interesting to note that continuity has been observed an ideal property of a mapping but, in certain instances, it may not be continuous at some points of the domain of definition.

In fixed point theorems, the assumption of continuity of mappings play a crucial role. However, there are several fixed point results which do not require the mapping to be continuous in the entire domain but at the fixed point the mappings are necessarily continuous, for instance, see [7] in details. Moreover, many researchers have studied diverse weaker assumptions of the continuity. Recently, Bisht [1] has explored finely the comparison of weaker forms of continuity in details and also categorize several contractive definitions which ensure the existence of the fixed point, even though the mappings may not be continuous at the fixed point with or without some weaker forms of continuity.

In 2019, Górnicki [4] obtained the fixed point result for asymptotically regular continuous mapping.

**Theorem 1.** *If  $(X, d)$  is a complete metric space and  $f : X \rightarrow X$  is a continuous asymptotically regular mapping and if there exist  $0 \leq M < 1$  and  $0 \leq K < +\infty$  satisfying*

$$(1) \quad d(fx, fy) \leq M d(x, y) + K \{d(x, fx) + d(y, fy)\}$$

*for all  $x, y \in X$ , then  $f$  has a unique fixed point  $p \in X$  and  $f^n x \rightarrow p$  for each  $x \in X$ .*

Further, Bisht [2] showed that the assumption of continuity considered in Theorem 2.6 of [4] can be relaxed to some weaker notions of continuity, viz. orbital continuity or  $k$ -continuity, which extends the scope of the study of fixed point theorems from the class of continuous mappings to a wider class of discontinuous mappings. The theorem due to Bisht [2] is as follows:

**Theorem 2** ([2]). *If  $(X, d)$  is a complete metric space and  $T : X \rightarrow X$  is an asymptotically regular mapping and if there exists  $0 \leq M < 1$  and  $0 \leq K < \infty$  satisfying*

$$(2) \quad d(Tx, Ty) \leq M d(x, y) + K \{d(x, Tx) + d(y, Ty)\}$$

*for all  $x, y \in X$ , then  $T$  has a unique fixed point  $p \in X$  provided  $T$  is either  $k$ -continuous for  $k \geq 1$  or orbitally continuous.*

Let  $\mathcal{S}$  denote the class of functions  $\alpha : [0, \infty) \rightarrow [0, 1)$  satisfying the condition:

$$\lim_{n \rightarrow \infty} \alpha(t_n) = 0 \Rightarrow \lim_{n \rightarrow \infty} t_n = 0.$$

Let  $\mathcal{U}$  denote the class of functions  $\varphi : [0, \infty) \rightarrow [0, \infty)$  satisfying:

- (i)  $\varphi(t) < t$  for all  $t > 0$ .
- (ii)  $\varphi$  is upper semi-continuous, i.e.,  
if  $t_n \rightarrow t \geq 0$ , then  $\limsup_{n \rightarrow \infty} \varphi(t_n) \geq \varphi(t)$ .

Using the above class of functions, Górnicki [3] obtained the following results for orbitally continuous or  $k$ -continuous mappings.

**Theorem 3** (Theorem 2.1 of Górnicki [3]). *Let  $(X, d)$  be a complete metric space and  $T : X \rightarrow X$  an asymptotically regular mapping. Suppose there exists  $\alpha \in \mathcal{S}, 0 \leq K < \infty$ , such that for each  $x, y \in X$ ,*

$$(3) \quad d(Tx, Ty) \leq \alpha(d(x, y))d(x, y) + K\{d(x, Tx) + d(y, Ty)\}.$$

*If  $T$  is  $k$ -continuous for some  $k \geq 1$  or  $T$  is orbitally continuous, then  $T$  has a unique fixed point  $z \in X$  and for each  $x \in X, T^n x \rightarrow z$  as  $n \rightarrow \infty$ .*

**Theorem 4** (Theorem 2.2 of Górnicki [3]). *Let  $(X, d)$  be a complete metric space and  $T : X \rightarrow X$  an asymptotically regular mapping. Suppose there exists  $\varphi \in \mathcal{U}, 0 \leq K < \infty$ , such that for each  $x, y \in X$ ,*

$$(4) \quad d(Tx, Ty) \leq \varphi(d(x, y)) + K\{d(x, Tx) + d(y, Ty)\}.$$

*If  $T$  is  $k$ -continuous for some  $k \geq 1$  or  $T$  is orbitally continuous, then  $T$  has a unique fixed point  $z \in X$  and for each  $x \in X, T^n x \rightarrow z$  as  $n \rightarrow \infty$ .*

**Remark 3.** Theorem 3 is a particular case of Theorem 4.

Khan and Oyetunbi [6] extended Theorem 1 for a pair of asymptotically regular self-mappings.

**Theorem 5** ([6]). *Suppose that  $(X, d)$  is a complete metric space,  $T$  and  $S$  are asymptotically regular self-mappings on  $X$  satisfying the following*

$$(5) \quad d(Tx, Sy) \leq Md(x, y) + K\{d(x, Tx) + d(y, Sy)\} \text{ for all } x, y \in X,$$

*where  $0 \leq M < 1$  and  $0 \leq K < \infty$ . Suppose further that  $T$  and  $S$  are either  $k$ -continuous for some  $k \geq 1$  or orbitally continuous. Then,  $T$  and  $S$  have a unique fixed point  $p$ . Additionally,  $\lim_{n \rightarrow \infty} T^n x = p = \lim_{n \rightarrow \infty} S^n x$  for any  $x \in X$ .*

Thereafter, Nguyen [8] established the following result for  $T$ -orbitally lower semi-continuous.

**Theorem 6** (Corollary 1 of [8]). *Let  $(X, d)$  be a complete metric space and  $T : X \rightarrow X$ . Assume that there exists  $x_0 \in X$  such that  $T$  is asymptotically regular at  $x_0$  and there exist  $\varphi \in \mathcal{U}, K \geq 0, l_1, l_2 > 0$  such that*

$$(6) \quad d(Tx, Ty) \leq \varphi(d(x, y)) + K \left\{ [d(x, Tx)]^{l_1} + [d(y, Ty)]^{l_2} \right\},$$

*for all  $x, y \in X$ . If  $x \rightarrow d(x, Tx)$  is  $T$ -orbitally lower semi-continuous then  $T$  has a unique fixed point  $z \in X$  and  $\lim_{n \rightarrow \infty} T^n x = z$  for all  $x \in O(T, x_0)$ .*

Now it is worthwhile to consider the notion of weakly orbital continuity or asymptotic continuity which are to be employed on some recent theorems established by Khan and Oyetunbi [6], Górnicki [3, 4], Bisht [2] and Nguyen [8]. Even though the mappings are not necessarily orbital continuous or  $k$ -continuous.

## 2. MAIN RESULTS

**Theorem 7.** *Theorem 2 holds provided  $T$  is either weak orbitally continuous or asymptotically continuous.*

*Proof.* Let  $x_0 \in X$ . Define a sequence  $\{x_n\}$  in  $X$  given by the rule  $x_{n+1} = Tx_n = T^n x_0$ . Then following the proof of Theorem 2.1 in [2], we conclude that  $\{x_n\}$  is a Cauchy sequence. Since  $X$  is complete, there exists a point  $z \in X$  such that  $\lim_{n \rightarrow \infty} x_n = z$ , and so  $\lim_{n \rightarrow \infty} Tx_n = z$ . Furthermore, for each  $k \geq 1$  we have  $\lim_{n \rightarrow \infty} T^k x_n = z$ .

Now let us suppose that  $T$  is weak orbitally continuous. Then there exists  $y_0 \in X$  such that  $\lim_{n \rightarrow \infty} T^n y_0 = z$  and  $\lim_{n \rightarrow \infty} T^{n+1} y_0 = Tz$ . This implies  $z = Tz$ . Hence  $z$  is a fixed point of  $T$ .

Finally, suppose that  $T$  is asymptotically continuous. Since  $\lim_{n \rightarrow \infty} x_n = z$  and  $\lim_{n \rightarrow \infty} T^k x_n = z$  for each  $k \geq 1$ , asymptotic continuity implies that  $\lim_{k, n \rightarrow \infty} T(T^k x_n) = Tz$ . This yields  $Tz = z$  because the following holds:

$$\lim_{k, n \rightarrow \infty} T(T^k x_n) = \lim_{k, n \rightarrow \infty} T^{k+1} x_n = z.$$

Hence  $z$  is a fixed point of  $T$ . □

**Remark 4.** Theorem 7 also a generalization of the result due to Górnicki [4].

Next we show that Theorem 2.1 and Theorem 2.2 of Górnicki [3] also hold for weak orbitally continuous mappings.

**Theorem 8.** *Theorems 3, 4 and 6 hold provided  $T$  is either weak orbitally continuous or asymptotically continuous.*

*Proof.* Let  $x_0 \in X$ . Define a sequence  $\{x_n\}$  in  $X$  given by the rule  $x_{n+1} = Tx_n = T^n x_0$ . Then following Theorems 2.1-2.2 in Górnicki [3] and Theorem 3 in Nguyen [8], we have  $\{x_n\}$  is a Cauchy sequence. Since  $X$  is complete, there exists a point  $z \in X$  such that  $\lim_{n \rightarrow \infty} x_n = z$ , and so  $\lim_{n \rightarrow \infty} Tx_n = z$ . Furthermore, for each  $k \geq 1$  we have  $\lim_{n \rightarrow \infty} T^k x_n = z$ .

Now suppose that  $T$  is weak orbitally continuous, then there exists  $y_0 \in X$  such that  $T^n y_0 \rightarrow z$  and  $T^{n+1} y_0 \rightarrow Tz$ . This implies  $z = Tz$ . Hence  $z$  is a fixed point of  $T$ .

Furthermore, suppose that  $T$  is asymptotically continuous. Since  $\lim_{n \rightarrow \infty} x_n = z$  and  $\lim_{n \rightarrow \infty} T^k x_n = z$  for each  $k \geq 1$ , asymptotic continuity implies that  $\lim_{k, n \rightarrow \infty} T(T^k x_n) = Tz$ . This yields  $Tz = z$  because the following holds:

$$\lim_{k, n \rightarrow \infty} T(T^k x_n) = \lim_{k, n \rightarrow \infty} T^{k+1} x_n = z.$$

Hence  $z$  is a fixed point of  $T$ . □

**Theorem 9.** *Theorem 5 holds if  $T$  and  $S$  are either weak orbitally continuous or asymptotically continuous.*

*Proof.* The proof of first two steps (Step 1 and Step 2) is similar to that of Theorem 2.2 in [6]. The next step is the following:

Step 3. Suppose that  $T$  is weak orbitally continuous. Since  $\{T^n x_0\}$  converges to  $p$  for each  $x_0$  in  $X$ , weakly orbital continuity implies that there exists  $y_0$  in  $X$  such that  $\lim_{n \rightarrow \infty} T^n y_0 = p$  and  $\lim_{n \rightarrow \infty} T^{n+1} y_0 = Tp$ . This implies  $p = Tp$ , that is,  $p$  is a fixed point of  $T$ . Similarly, suppose that  $S$  is weak orbitally continuous. Since  $\{S^n x_0\}$  converges to  $p$  for each  $x_0$  in  $X$ , weakly orbital continuity implies that there exists  $z_0$  in  $X$  such that  $\lim_{n \rightarrow \infty} S^n z_0 = p$  and  $\lim_{n \rightarrow \infty} S^{n+1} z_0 = Sp$ . This implies  $p = Sp$ , that is,  $p$  is a fixed point of  $S$ . Hence  $p$  is a common fixed point of  $T$  and  $S$ .

Finally, we suppose that  $T$  is asymptotically continuous. Let  $x_n = T^n x_0$  for any  $x_0$  in  $X$ . Then, by Step 3, we get  $\lim_{n \rightarrow \infty} x_n = p$  and so  $\lim_{n \rightarrow \infty} T^k x_n = p$  for each  $k \geq 1$ . Therefore, asymptotic continuity implies that  $\lim_{k, n \rightarrow \infty} T(T^k x_n) = Tp$ . This yields  $Tp = p$  because the following holds:

$$\lim_{k, n \rightarrow \infty} T(T^k x_n) = \lim_{k, n \rightarrow \infty} T^{k+1} x_n = p.$$

Hence  $p$  is a fixed point of  $T$ .

Similarly, we can show that  $p$  is a fixed point of  $S$  whenever we suppose the mapping  $S$  is asymptotically continuous. So  $p$  is a common fixed point of  $T$  and  $S$ . Furthermore, the uniqueness of common fixed point is similar to that of [6].  $\square$

The following examples illustrate our results.

**Example 2** (see also [8]). Let  $X = \{0, 1\} \cup \{\frac{1}{2^n} : n = 1, 2, \dots\} \cup \{1 + \frac{1}{2^n} : n = 1, 2, \dots\}$  be the metric space with usual metric. Then  $(X, d)$  is a complete metric space. Let  $T : X \rightarrow X$  be defined by

$$Tx = \begin{cases} 0, & \text{if } x = 0; \\ \frac{3}{2}, & \text{if } x = 1; \\ 1 + \frac{1}{2^{n+1}}, & \text{if } x = \frac{1}{2^n}, n = 1, 2, \dots; \\ \frac{1}{2^n}, & \text{if } x = 1 + \frac{1}{2^n}, n = 1, 2, \dots \end{cases}$$

Then  $T$  is neither orbitally continuous nor  $k$ -continuous for any  $k$ . To see, take a sequence  $\{x_n\}$  defined by  $x_n = \frac{1}{2^n}$ . Then we get  $x_n \rightarrow 0$  as  $n \rightarrow \infty$  and  $Tx_n = 1 + \frac{1}{2^{n+1}} \rightarrow 1 \neq T0$ . Therefore  $T$  is not orbitally continuous. Let  $\{a_n\}$  be a sequence in  $X$  with  $a_n = \frac{1}{2^n}$  for  $n \geq 1$ . For  $i = 1, 2, \dots$  we have  $\lim_{n \rightarrow \infty} T^{2i-1} a_n = 1$  and  $\lim_{n \rightarrow \infty} T^{2i} a_n = 0 \neq \frac{3}{2} = T1$ , and

$\lim_{n \rightarrow \infty} T^{2i+1}a_n = 1 \neq T0$ . That is,  $T$  is neither  $2i$ -continuous nor  $(2i + 1)$ -continuous. Thus  $T$  is not  $k$ -continuous for any  $k \geq 1$ . Moreover,

- (i)  $T$  is asymptotically regular at 0.
- (ii) If we take  $x_n = 0$  then  $Tx_n \rightarrow 0 = T0$  and hence,  $T$  is weak orbitally continuous.
- (iii) Conditions (1) and (2) are satisfied with  $M = \frac{1}{2}$  and  $K = 10$ ; condition (4) is satisfied with  $\varphi(t) = \frac{t}{2} \in \mathcal{U}, K = 10$ . We have  $d(Tx, Ty) \leq \frac{3}{2}$  for all  $x, y \in X$ . For  $x = y$ , conditions (1), (2) and (4) are obvious. For  $x, y \neq 0$  and  $x \neq y$ , we have

$$d(Tx, Ty) \leq \frac{3}{2} < 5 \leq 10[d(x, Tx) + d(y, Ty)]$$

that is, conditions (1), (2), (4) and (6) hold.

Thus all the hypothesis of Theorem 7 and 8 are satisfied and  $T$  has a unique fixed point 0. However, Theorem 1 (Theorem 2.6 of [4]), Theorem 2 (Theorem 2.1 of [2]), Theorems 3 and 4 (Theorems 2.1 and 2.2 of [3]) are not satisfied because  $T$  is not orbitally or  $k$ -continuous. Although Theorem 5 (Theorem 2.2 of [6]) is not satisfied even for  $S = T$ .

**Example 3.** Let  $X = [0, \infty)$  be equipped with Euclidean metric. Let  $T : X \rightarrow X$  be defined by

$$Tx = \begin{cases} 0, & \text{if } 0 \leq x < 1; \\ \frac{1}{2}, & \text{if } x = 1; \\ \frac{x}{3}, & \text{if } x > 1. \end{cases}$$

Then  $T$  is neither  $k$ -continuous nor  $f^k$ -continuous for any  $k$ . To see, consider a sequence  $\{x_n\}$  in  $X$  defined by  $x_n = 3^{k-1} + \frac{1}{3n}$  where  $k$  is a natural number greater than 1. Then we get  $\lim_{n \rightarrow \infty} T^{k-1}x_n = \lim_{n \rightarrow \infty} (1 + \frac{1}{3^{k-1}n}) = 1$  and  $\lim_{n \rightarrow \infty} T^k x_n = \lim_{n \rightarrow \infty} (\frac{1}{3} + \frac{1}{3^{k+1}n}) = \frac{1}{3} \neq T1$ , hence  $T$  is not  $k$ -continuous.

Also we have  $\lim_{n \rightarrow \infty} x_n = 3^{k-1}$  and  $\lim_{n \rightarrow \infty} T^k x_n = \frac{1}{3} \neq T^k(3^{k-1})$ , this shows that  $T$  is not  $T^k$ -continuous. Indeed,  $T$  is asymptotically continuous because  $\lim_{k \rightarrow \infty} T^k x = 0$  for each  $x \in X$ , and so we have  $\lim_{k, n \rightarrow \infty} T^k x_n = 0$  and

$\lim_{k, n \rightarrow \infty} T(T^k x_n) = 0 = T0$ . However, we have the following:

- (i)  $T$  is asymptotically regular.
- (ii) Conditions (1) and (2) are satisfied with  $M = \frac{1}{2}$  and  $K = 4$ , and conditions (4) and (6) are also satisfied with  $l_1 = l_2 = 1, \varphi(t) = \frac{t}{2} \in \mathcal{U}, K = 4$ .

Hence all the hypothesis of Theorem 7 and 8 are satisfied and  $T$  has a unique fixed point 0. Although, Theorem 1 (Theorem 2.6 of [4]), Theorem 2 (Theorem 2.1 of [2]), Theorems 3 and 4 (Theorems 2.1-2.2 of [3]) are not satisfied because  $T$  is  $k$ -continuous.

**Example 4.** Let  $X = \{0, 1\} \cup \{\frac{1}{2^n} : n = 1, 2, \dots\} \cup \{1 + \frac{1}{2^n} : n = 1, 2, \dots\}$  be the metric space with usual metric. Then  $(X, d)$  is a complete metric space. Let  $T, S : X \rightarrow X$  be defined by

$$Tx = \begin{cases} 0, & \text{if } x = 0; \\ \frac{5}{4}, & \text{if } x = 1; \\ 1 + \frac{1}{2^{n+1}}, & \text{if } x = \frac{1}{2^n}; \\ \frac{1}{2^n}, & \text{if } x = 1 + \frac{1}{2^n}, \end{cases} \quad \text{and} \quad Tx = \begin{cases} 0, & \text{if } x = 0; \\ \frac{9}{8}, & \text{if } x = 1; \\ 1 + \frac{1}{2^{n+1}}, & \text{if } x = \frac{1}{2^n}; \\ \frac{1}{2^n}, & \text{if } x = 1 + \frac{1}{2^n}, \end{cases}$$

where  $n = 1, 2, 3, \dots$

Following the analogous steps of Example 2, we have  $T$  and  $S$  are neither orbitally continuous nor  $k$ -continuous for any  $k$ . Moreover,

- (i)  $T$  and  $S$  are asymptotically regular at 0.
- (ii)  $T$  and  $S$  are weak orbitally continuous.
- (iii) Condition (5) is satisfied with  $M = \frac{1}{2}$  and  $K = 10$ .

Hence all the hypothesis of Theorem 9 are satisfied and  $T$  and  $S$  have a unique common fixed point 0. However, Theorem 5 (Theorem 2.2 of [6]) is not satisfied because  $T$  and  $S$  are not orbitally or  $k$ -continuous.

#### COMPLIANCE WITH ETHICAL STANDARDS

**Conflicts of interest.** The authors declare that they have no conflict of interest.

**Ethical approval.** This article does not contain any studies with human participants or animals performed by any of the authors.

**Informed consent.** Informed consent was obtained from all individual participants included in the study.

#### ACKNOWLEDGMENTS

The authors are thankful to the learned referees for their useful suggestions for the improvement of the paper and removing certain obscurities in the presentation.

#### REFERENCES

- [1] R. K. Bisht, *An overview of the emergence of weaker continuity notions, various classes of contractive mappings and related fixed point theorems*, Journal of Fixed Point Theory and Applications, 25 (2023), Article ID: 11, 1-29.
- [2] R. K. Bisht, *A note on the fixed point theorem of Górnicki*, Journal of Fixed Point Theory and Applications, 21 (2019), Article ID: 54, 1-3.
- [3] J. Górnicki, *On some mappings with a unique fixed point*, Journal of Fixed Point Theory and Applications, 22 (2020), Article ID: 8, 1-7.
- [4] J. Górnicki, *Remarks on asymptotic regularity and fixed points*, Journal of Fixed Point Theory and Applications, 21 (2019), Article ID: 29, 1-20.

- 
- [5] T. L. Hicks, B. E. Rhoades, *A Banach type fixed-point theorem*, Math. Japon. 24 (1979/80), 327-330.
- [6] A. R. Khan, D. M. Oyetunbi, *On some mappings with a unique common fixed point*, Journal of Fixed Point Theory and Applications, 22:47 (2020), Article ID: 47, 1-7.
- [7] V. Mainali, K. Dani, N. Chandra, M. C. Joshi, *Generalized rational type contractions and discontinuity at fixed point*, Journal of Advanced Mathematical Studies, 18 (1) (2025), 24-33.
- [8] L. V. Nguyen, *On fixed points of asymptotically regular mappings*, Rendiconti del Circolo Matematico di Palermo Series 2, 70 (2021), 709-719.
- [9] A. Pant, R. P. Pant, M. C. Joshi, *Caristi type and Meir-Keeler type fixed point theorems*, Filomat, 33 (12) (2019), 3711-3721.
- [10] R. P. Pant, V. Rakočević, *Fixed point and periodic point theorems*, Acta Scientiarum Mathematicarum, 90 (2024), 175-192.

**BHARTI JOSHI**

DEPARTMENT OF MATHEMATICS  
DSB CAMPUS, KUMAUN UNIVERSITY  
NAINITAL-263002, UTTARAKHAND  
INDIA

*E-mail address:* bhartijoshi20592@gmail.com

**NAVEEN CHANDRA**

DEPARTMENT OF MATHEMATICS  
BGR CAMPUS, HNB GARHWAL UNIVERSITY  
PAURI-246001, UTTARAKHAND  
INDIA

*E-mail address:* cnaveen329@gmail.com