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Relation between b-metric and fuzzy metric spaces

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Abstract. In this work we have considered several common fixed point results in b-metric spaces for weak compatible mappings. By applications of these results we establish some fixed point theorems in b-fuzzy metric spaces.

1. Introduction

In this paper we establish some fixed point results in a b-fuzzy metric space by applications of certain fixed point theorems in b-metric spaces. Also we prove some fixed point results in b-metric spaces. Fuzzy metric space was first introduced by Kramosil and Michalek [3]. Subsequently, George and Veeramani had given a modified definition of fuzzy metric spaces [1]. Fixed point results in such spaces have been established in a large number of works. Some of these works are noted in [2, 4, 5, 7, 10, 11].

Definition 1.1. [1] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a * 1 = a$, for all $a \in [0,1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 1.2. [1] A 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions, for each $x, y, z \in X$ and $t, s > 0$:

(1) $M(x, y, t) > 0$,

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(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(5) $M(x, y, .) : (0, \infty) \to [0, 1]$ is continuous.

Definition 1.3. [8, 9] A 3-tuple $(X, M, *)$ is called a b-fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions, for each $x, y, z \in X$, $t, s > 0$ and a given real number $b \geq 1$:

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, \frac{t}{b}) * M(y, z, \frac{s}{b}) \leq M(x, z, t + s)$,
(5) $M(x, y, .) : (0, \infty) \to [0, 1]$ is continuous.

We present an example shows that a b-fuzzy metric on X need not be a fuzzy metric on X.

Example 1.4. Let $M(x, y, t) = e^{\frac{-|x-y|^p}{t}}$, where $p > 1$ is a real number. We show that M is a b-fuzzy metric with $b = 2^{p-1}$.

Obviously conditions (1), (2), (3) and (5) of Definition 1.3 are satisfied.

If $1 < p < \infty$, then the convexity of the function $f(x) = x^p$ $(x > 0)$ implies

$$\left(\frac{a+c}{2}\right)^p \leq \frac{1}{2}\left(a^p + c^p\right),$$

and hence, $(a + c)^p \leq 2^{p-1}(a^p + c^p)$ holds. Therefore,

$$\begin{aligned} \frac{|x-y|^p}{t+s} &\leq 2^{p-1}\frac{|x-z|^p}{t+s} + 2^{p-1}\frac{|z-y|^p}{t+s} \\ &\leq 2^{p-1}\frac{|x-z|^p}{t} + 2^{p-1}\frac{|z-y|^p}{s} \\ &= \frac{|x-z|^p}{t/2^{p-1}} + \frac{|z-y|^p}{s/2^{p-1}}. \end{aligned}$$

Thus for each $x, y, z \in X$ we obtain

$$\begin{aligned} M(x, y, t+s) &= e^{\frac{-|x-y|^p}{t+s}} \\ &\geq M(x, z, \frac{t}{2^{p-1}}) * M(z, y, \frac{s}{2^{p-1}}), \end{aligned}$$

where $a * b = ab$. So condition (4) of Definition 1.3 hold and M is a b-fuzzy metric.

It should be noted that in preceding example, for $p = 2$ it is easy to see that $(X, M, *)$ is not a fuzzy metric space.

Example 1.5. Let $M(x,y,t)=e^{\frac{-d(x,y)}{t}}$ or $M(x,y,t)=\frac{t}{t+d(x,y)}$, where d is a b-metric on X and $a*c=ac$, for all $a,c\in[0,1]$. Then it is easy to show that M is a b-fuzzy metric.

Obviously conditions (1), (2), (3) and (5) of Definition 1.3 are satisfied. For each $x,y,z\in X$ we obtain

$$\begin{aligned} M(x,y,t+s) &= e^{\frac{-d(x,y)}{t+s}} \\ &\geq e^{-b\frac{d(x,z)+d(z,y)}{t+s}} \\ &= e^{-b\frac{d(x,z)}{t+s}}\cdot e^{-b\frac{d(z,y)}{t+s}} \\ &\geq e^{\frac{-d(x,z)}{t/b}}\cdot e^{\frac{-d(z,y)}{s/b}} \\ &= M(x,z,\frac{t}{b})*M(z,y,\frac{s}{b}). \end{aligned}$$

So condition (4) of Definition 1.3 is hold and M is a b-fuzzy metric. Similarly, it is easy to see that $M(x,y,t)=\frac{t}{t+d(x,y)}$ is a b-fuzzy metric.

2. MAIN RESULTS

Lemma 2.1. *Let $(X,M,*)$ be a b-fuzzy metric space with $a*c\geq ac$, for all $a,c\in[0,1]$. If $d:X^2\to[0,\infty)$ is defined by $d(x,y)=\lim_{\epsilon\to 0}\int_\epsilon^1\log_\alpha M(x,y,t)dt$, for $0<\alpha<1$, then d is an $2b$-metric on X.*

Proof. By definition, we have that $d(x,y)$ is well defined for each $x,y\in X$. Clearly, $d(x,y)\geq 0$, for all $x,y\in X$. Moreover, $d(x,y)=0$ if and only if $\log_\alpha(M(x,y,t))=0$ if and only if $M(x,y,t)=1$ if and only if $x=y$.

Since

$$\begin{aligned} M(x,y,t) &\geq M(x,z,\frac{t}{2b})*M(z,y,\frac{t}{2b}) \\ &\geq M(x,z,\frac{t}{2b})\cdot M(z,y,\frac{t}{2b}), \end{aligned}$$

it follows that

$$\begin{aligned} d(x,y) &= \lim_{\epsilon\to 0}\int_\epsilon^1 \log_\alpha M(x,y,t)dt \\ &\leq \lim_{\epsilon\to 0}\int_\epsilon^1 \log_\alpha M(x,z,\frac{t}{2b})\cdot M(z,y,\frac{t}{2b})dt \\ &\leq \lim_{\epsilon\to 0}\int_\epsilon^1 \log_\alpha M(x,z,\frac{t}{2b})dt+\lim_{\epsilon\to 0}\int_\epsilon^1 \log_\alpha M(z,y,\frac{t}{2b})dt \\ &= 2b\lim_{\epsilon\to 0}\int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} \log_\alpha M(x,z,t)dt+2b\lim_{\epsilon\to 0}\int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} \log_\alpha M(z,y,t)dt \\ &\leq 2b\left[\lim_{\epsilon\to 0}\int_{\frac{\epsilon}{2b}}^{1} \log_\alpha M(x,z,t)dt+\lim_{\epsilon\to 0}\int_{\frac{\epsilon}{2b}}^{1} \log_\alpha M(x,z,t)dt\right] \end{aligned}$$

$$= \quad 2b[d(x,z)+d(z,y)].$$

This proves that d is an $2b$-metric on X. □

The following lemma plays an important role to give fixed point results on a fuzzy metric space.

Lemma 2.2. *Let $(X, M, *)$ be a b-fuzzy metric space with $a * c \geq ac$, for all $a, c \in [0,1]$. If $d : X^2 \to [0,\infty)$ is define by $d(x,y) = \lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha M(x,y,t)dt$, for all $0 < \alpha < 1$, then:*

(1) *$\{x_n\}$ is a Cauchy sequence in b-fuzzy metric $(X, M, *)$ if and only if it is a Cauchy sequence in the $2b$- metric space (X, d).*
(2) *A b-fuzzy metric space $(X, M, *)$ is complete if and only if the $2b$-metric space (X, d) is complete.*

Proof. First we show that every Cauchy sequence in $(X, M, *)$ is a Cauchy sequence in (X, d). To this end let $\{x_n\}$ be a Cauchy sequence in $(X, M, *)$. Then $\lim_{n,m\to\infty} M(x_n, x_m, t) = 1$. Since

$$d(x_n, x_m) = \lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha M(x_n, x_m, t)dt,$$

is a $2b$-metric. Hence, we have

$$\begin{aligned}\lim_{n,m\to\infty} d(x_n, x_m) &= \lim_{n,m\to\infty}\lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha M(x_n, x_m, t)dt \\ &= \lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha \lim_{n,m\to\infty} M(x_n, x_m, t)dt = 0,\end{aligned}$$

so, we conclude that $\{x_n\}$ is a Cauchy sequence in (X, d).

Next we prove that completeness of (X, d) implies completeness of $(X, M, *)$. Indeed, if $\{x_n\}$ is a Cauchy sequence in $(X, M, *)$ then it is also a Cauchy sequence in (X, d). Since the $2b$-metric space (X, d) is complete we deduce that there exists $y \in X$ such that $\lim_{n\to\infty} d(x_n, y) = 0$. Therefore,

$$\lim_{n\to\infty}\lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha M(x_n, y, t)dt \quad = \quad \lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha \lim_{n\to\infty} M(x_n, y, t)dt = 0,$$

that is $\lim_{n\to\infty} M(x_n, y, t)dt = 1$. Hence we follow that $\{x_n\}$ is a convergent sequence in $(X, M, *)$.

Now we prove that every Cauchy sequence $\{x_n\}$ in (X, d) is a Cauchy sequence in $(X, M, *)$. Since $\{x_n\}$ is a Cauchy sequence in (X, d), then

$$\begin{aligned}\lim_{n,m\to\infty} d(x_n, x_m) &= \lim_{n,m\to\infty}\lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha M(x_n, x_m, t)dt \\ &= \lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha \lim_{n,m\to\infty} M(x_n, x_m, t)dt = 0.\end{aligned}$$

Hence, $\lim_{n,m\to\infty} M(x_n, x_m, t) = 1$.

That is, $\{x_n\}$ is a Cauchy sequence in $(X, M, *)$.

We will establish the lemma if we prove that (X, d) is complete if so is $(X, M, *)$. Let $\{x_n\}$ be a Cauchy sequence in (X, d). Then $\{x_n\}$ is a Cauchy sequence in $(X, M, *)$, and so it is convergent to a point $y \in X$ with

$$\lim_{n\to\infty} M(x_n, y, t) = 1.$$

As a consequence we have

$$\begin{aligned}\lim_{n\to\infty} d(x_n, y) &= \lim_{n\to\infty}\lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha M(x_n, y, t)dt \\ &= \lim_{\epsilon\to 0} \int_\epsilon^1 \log_\alpha \lim_{n\to\infty} M(x_n, y, t)dt = 0.\end{aligned}$$

Therefore (X, d) is complete. □

Lemma 2.3. *Let $(X, M, *)$ be a b-fuzzy metric space with $a * c = \min\{a, c\}$, for all $a, c \in [0, 1]$. We define $d : X^2 \to [0, \infty)$ by*

$$d(x, y) = \lim_{\epsilon\to 0} \int_\epsilon^1 \cot(\frac{\pi}{2} M(x, y, t))dt,$$

then d is an $2b$-metric on X.

Proof. Clearly, $d(x, y) \geq 0$, for all $x, y \in X$. Moreover, $d(x, y) = 0$ if and only if $\cot(\frac{\pi}{2} M(x, y, t)) = 0$ if and only if $M(x, y, t) = 1$ if and only if $x = y$.

Since,

$$M(x, y, t) \geq M(x, z, \frac{t}{2b}) * M(z, y, \frac{t}{2b}) = \min\{M(x, z, \frac{t}{2b}), M(z, y, \frac{t}{2b})\},$$

and also since $0 < \frac{\pi}{2} M(x, y, \frac{t}{2b}) \leq \frac{\pi}{2}$, it follows that,

$$\begin{aligned}
d(x, y) &= \lim_{\epsilon\to 0} \int_\epsilon^1 \cot(\frac{\pi}{2} M(x, y, t))dt \\
&\leq \lim_{\epsilon\to 0} \int_\epsilon^1 \cot[\frac{\pi}{2}(M(x, z, \frac{t}{2b}) * M(z, y, \frac{t}{2b}))]dt \\
&= 2b \left(\lim_{\epsilon\to 0} \int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} \cot(\frac{\pi}{2} \min\{M(x, z, t), M(z, y, t)\})dt \right) \\
&= 2b \min\left\{ \lim_{\epsilon\to 0} \int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} \cot(\frac{\pi}{2} M(x, z, t))dt, \lim_{\epsilon\to 0} \int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} \cot(\frac{\pi}{2} M(z, y, t))dt \right\} \\
&\leq 2b\lim_{\epsilon\to 0} \int_{\frac{\epsilon}{2b}}^1 \cot(\frac{\pi}{2} M(x, z, t))dt + 2b\lim_{\epsilon\to 0} \int_{\frac{\epsilon}{2b}}^1 \cot(\frac{\pi}{2} M(z, y, t))dt \\
&= 2b[d(x, z) + d(z, y)],
\end{aligned}$$

that is d is an $2b$-metric on X. □

Remark 2.4. Let $a, b \in (0,1]$, then it is a standard result that

$$\text{arccot}(\min\{a,b\}) \leq \text{arccot}(a) + \text{arccot}(b) - \frac{\pi}{4}.$$

Lemma 2.5. *Let $(X, M, *)$ be a $2b$-fuzzy metric space with $a*c = \min\{a,c\}$, for all $a, c \in [0,1]$. If we define $d : X^2 \to [0,\infty)$ by*

$$d(x,y) = \lim_{\epsilon \to 0} \int_{\epsilon}^{1} (\frac{4}{\pi} \text{arccot}(M(x,y,t)) - 1)dt,$$

then d is an $2b$-metric on X.

Proof. Clearly, $0 \leq d(x,y) < 1$, for all $x, y \in X$. Moreover, $d(x,y) = 0$ if and only if $\frac{4}{\pi}\text{arccot}(M(x,y,t)) - 1 = 0$ if and only if $\text{arccot}(M(x,y,t)) = \frac{\pi}{4}$ if and only if $M(x,y,t) = 1$ if and only if $x = y$. Since

$$M(x,y,t) \geq M(x,z,\frac{t}{2b}) * M(z,y,\frac{t}{2b}) = \min\{M(x,z,\frac{t}{2b}), M(z,y,\frac{t}{2b})\},$$

it follows that

$$\begin{aligned}
\text{arccot}(M(x,y,t)) &\leq \text{arccot}[M(x,z,\frac{t}{2b}) * M(z,y,\frac{t}{2b})] \\
&= \text{arccot}(\min\{M(x,z,\frac{t}{2b}), M(z,y,\frac{t}{2b})\}) \\
&\leq \text{arccot}(M(x,z,\frac{t}{2b})) + \text{arccot}(M(z,y,\frac{t}{2b})) - \frac{\pi}{2}.
\end{aligned}$$

Hence,

$$\begin{aligned}
d(x,y) &= \lim_{\epsilon \to 0} \int_{\epsilon}^{1} (\frac{4}{\pi} \text{arccot}(M(x,y,t)) - 1)dt \\
&\leq \lim_{\epsilon \to 0} \int_{\epsilon}^{1} (\frac{4}{\pi} \text{arccot}(M(x,z,\frac{t}{2b})) - 1)dt \\
&\qquad + \lim_{\epsilon \to 0} \int_{\epsilon}^{1} (\frac{4}{\pi} \text{arccot}(M(z,y,\frac{t}{2b})) - 1)dt \\
&= 2b\lim_{\epsilon \to 0} \int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} (\frac{4}{\pi} \text{arccot}(M(x,z,t)) - 1)dt \\
&\qquad + 2b\lim_{\epsilon \to 0} \int_{\frac{\epsilon}{2b}}^{\frac{1}{2b}} (\frac{4}{\pi} \text{arccot}(M(z,y,t)) - 1)dt \\
&\leq 2b \left(\lim_{\epsilon \to 0} \int_{\frac{\epsilon}{2b}}^{1} (\frac{4}{\pi} \text{arccot}(M(x,z,t)) - 1)dt \right. \\
&\qquad \left. + \lim_{\epsilon \to 0} \int_{\frac{\epsilon}{2b}}^{1} (\frac{4}{\pi} \text{arccot}(M(z,y,t)) - 1)dt \right) \\
&= 2b[d(x,z) + d(z,y)],
\end{aligned}$$

that is d is an $2b$-metric on X. ☐

Remark 2.6. Let $(X, M, *)$ be a fuzzy metric space with $a * c \geq ac$, for all $a, c \in [0,1]$. If sequence $\{x_n\}$ in X converges to x, that is, for every $0 < \epsilon < 1$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$, for all $n \geq n_0$ and each $t > 0$, then $d(x_n, x) \to 0$ where $d(x, y) = \lim_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(x, y, t)dt$. Also it is a Cauchy sequence if for each $0 < \epsilon < 1$ and $t > 0$, there exits $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$. It follows that $d(x_n, x_m) = \lim_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(x_n, x_m, t)dt < \lim_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha (1 - \epsilon)dt < \eta$, for every $\eta = (1 - \alpha) \log_\alpha (1 - \epsilon)$. Thus $\{x_n\}$ in $2b$-metric (X, d) is a Cauchy sequence.

Theorem 2.7. [6] *Suppose that f, g, S and T are self mappings of a complete b-metric space (X, d), with $f(X) \subseteq T(X)$, $g(X) \subseteq S(X)$ and that the pairs $\{f, S\}$ and $\{g, T\}$ are compatible. If*

$$d(fx, gy) \leq \frac{q}{b^4} \; max\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}(d(Sx, gy)+d(fx, Ty))\}, \tag{2.1}$$

for each $x, y \in X$, with $0 < q < 1$. Then f, g, S and T have a unique common fixed point in X provided that S and T are continuous.

We next apply theorem 2.7 to establish the following theorem in fuzzy metric spaces.

Theorem 2.8. *Let $(X, M, *)$ be a complete fuzzy metric space with $a*c \geq ac$ for all $a, c \in [0,1]$. Let f, g, S and T be self mappings on X with $f(X) \subseteq T(X)$, $g(X) \subseteq S(X)$ and that the pairs $\{f, S\}$ and $\{g, T\}$ are compatible. If there exists $q \in (0,1)$ such that for each $x, y \in X$,*

$$M(fx, gy, t) \geq \min \begin{pmatrix} M(Sx, Ty, t), M(fx, Sx, t), \\ M(gy, Ty, t), \sqrt{M(Sx, gy, t) \cdot M(fx, Ty, t))} \end{pmatrix}^{\frac{q}{(2b)^4}}$$

If S and T are continuous, then f, g, S and T have a unique common fixed point in X.

Proof. We define $d(x, y) = \lim_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(x, y, t)dt$ for every $x, y \in X$ where $0 < \alpha < 1$. Then by Lemma 2.1 and Lemma 2.2 (X, d) is a complete $2b-$metric space. From the above inequality, we get,

$$\lim_{\epsilon \to 0} \int_{\epsilon}^{1} \log_\alpha M(fx, gy, t)dt \leq$$

$$\frac{q}{(2b)^4} \max \left(\begin{array}{l} \lim\limits_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(Sx, Ty, t)dt, \lim\limits_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(fx, Sx, t)dt, \\ \lim\limits_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(gy, Ty, t)dt, \\ \frac{1}{2}(\lim\limits_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(Sx, gy, t)dt + \lim\limits_{\epsilon \to 0} \int_\epsilon^1 \log_\alpha M(fx, Ty, t)dt) \end{array} \right),$$

which is,

$$d(fx, gy) \leq \frac{q}{(2b)^4} \max \left(\begin{array}{l} d(Sx, Ty), d(fx, Sx), \\ d(gy, Ty), \frac{1}{2}(d(Sx, gy) + d(fx, Ty)) \end{array} \right).$$

Hence all the conditions of Theorem 2.7 hold, so the conclusion of Theorem 2.8 follows by an application of Theorem 2.7. □

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