MATHEMATICA MORAVICA
VOL. 2 (1998), 149–159

ON TOPOLOGICAL n-GROUPS

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Abstract. Let (Q, A) be an $n-$group, $^{-1}$ its inversing operation [: [13, 16], 1.3], $n \geq 2$ and let Q be equipped with a topology $\mathcal{O}$. Then, in this paper, we say that $(Q, A, \mathcal{O})$ is a topological $n-$group iff: a) the $n-$ary operation A is continuous in $\mathcal{O}$, and b) the $(n-1)-$ary operation $^{-1}$ is continuous in $\mathcal{O}$. The main result of the paper is the following proposition. Let (Q, A) be an $n-$group, $n \geq 3$ and let $(Q, \{\cdot, \varphi, b\})$ be an arbitrary $nHG-$ algebra associated to the $n-$group (Q, A) [: [15], 1.5]. Also, Q is equipped with a topology $\mathcal{O}$. Then, $(Q, A, \mathcal{O})$ is a topological $n-$ group iff the following statements hold: 1) $(Q, \cdot, \mathcal{O})$ is a topological group [:e.g. [7]], and 2) the unary operation φ is continuous in $\mathcal{O}$.

1. Preliminaries

1.1. Definition: *Let $n \geq 2$ and let (Q, A) be an $n-$groupoid. We say that (Q, A) is a Dörnte $n-$group [briefly: $n-$group] iff is an $n-$semigroup and an $n-$quasigroup as well [: see, e. g. [4–6]].*[1)]

1.2. Proposition [16]: *Let $n \geq 2$ and let (Q, A) be an $n-$groupoid. Then the following statements are equivalent: (i) (Q, A) is an $n-$group; (ii) there are mappings $^{-1}$ and $\mathbf{e}$ respectively of the sets Q^{n-1} and Q^{n-2} into the set Q such that the following laws hold in the algebra $(Q, \{A,^{-1}, \mathbf{e}\})$ [of the type $< n, n-1, n-2 >$]*

(a) $A\left(x_1^{n-2}, A(x_{n-1}^{2n-2}\right), x_{2n-1}) = A\left(x_1^{n-1}, A\left(x_n^{2n-1}\right)\right)$,

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AMS Mathematics Subject Classification 1991. Primary: 20N15, 22A30.

Key words and phrases: $n-$semigroups, $n-$quasigroups, $n-$groups, $\{1, n\}-$neutral operations on $n-$groupoids, inversing operation on $n-$group, $nHG-$algebras, topological groups.

[1)]A notion of an $n-$group was introduced by W. Dörnte in [1] as a generalization of the notion of a group.

(*b*) $A\left(\mathbf{e}\left(a_1^{n-2}\right), a_1^{n-2}, x\right) = x$ *and*

(*c*) $A\left(\left(a_1^{n-2}, a\right)^{-1}, a_1^{n-2}, a\right) = \mathbf{e}\left(a_1^{n-2}\right)$; *and*

(*iii*) *there are mappings* $^{-1}$ *and* $\mathbf{e}$ *respectively of the sets* Q^{n-1} *and* Q^{n-2} *into the set* Q *such that the following laws hold in the algebra* $(Q, \{A, ^{-1}, \mathbf{e}\})$ *[of the type* $< n, n-1, n-2 >$*]*

($\bar{a}$) $A\left(A\left(x_1^n\right), x_{n+1}^{2n-1}\right) = A\left(x_1, A\left(x_2^{n+1}\right), x_{n+2}^{2n-1}\right)$,

($\bar{b}$) $A\left(x, a_1^{n-2}, \mathbf{e}\left(a_1^{n-2}\right)\right) = x$ *and*

($\bar{c}$) $A\left(a, a_1^{n-2}, \left(a_1^{n-2}, a\right)^{-1}\right) = \mathbf{e}\left(a_1^{n-2}\right)$.

1.3. Remarks: $\mathbf{e}$ is an $\{1,n\}$−neutral operation of n−grupoid (Q, A) iff algebra $(Q, \{A, \mathbf{e}\})$ of type $< n, n-2 >$ satisfies the laws (b) and $(\bar{b})$ from 1.2 */*:[12]*/*. The notion of $\{i,j\}$−neutral operation $(i, j \in \{1, \ldots, n\}, i < j)$ of an n−groupoid is defined in a similar way */*:[12]*/*. Every n−groupoid there is **at most one** $\{i,j\}$−neutral operations */*:[12]*/*. In every n−group, $n \geq 2$, there is a $\{1,n\}$−neutral operation */*:[12]*/*. There are n−groups without $\{i,j\}$−neutral operations with $\{i,j\} \neq \{1,n\}$*/*:[14]*/*. In [14], n−groups with $\{i,j\}$−neutral operations, for $\{i,j\} \neq \{1,n\}$ are described. Operation $^{-1}$ from 1.2 */*$(c), (\bar{c})$*/* is a generalization of the inversing operation in a group. In fact, if (Q, A) is an n−group, $n \geq 2$, then for every $a \in Q$ and for every sequence a_1^{n-2} over Q is

$$\left(a_1^{n-2}, a\right)^{-1} \overset{def}{=} \mathsf{E}\left(a_1^{n-2}, a, a_1^{n-2}\right),$$

where E is an $\{1, 2n-1\}$−neutral operation of the $(2n-1)$−group $(Q, \overset{2}{A})$; $\overset{2}{A}(x_1^{2n-1}) \overset{def}{=} A(A(x_1^n), x_{n+1}^{2n-1})$ */*:[13]*/*. (For $n = 2$, $a^{-1} = \mathsf{E}(a)$; a^{-1} is the inverse element of the element a with respect to the neutral element $\mathbf{e}(\emptyset)$ of the group (Q, A).)

1.4. Proposition *(Hosszú-Gluskin Theorem)* **[2-3]**: *For every* n*−group* $(Q, A), n \geq 3$, *there is an algebra* $(Q, \{\cdot, \varphi, b\})$ *such that the following statements hold:* $1°(Q, \cdot)$ *is a group;* $2° \varphi \in Aut(Q, \cdot)$; $3°$ $\varphi(b) = b$; $4°$ *for every* $x \in Q$, $\varphi^{n-1}(x) \cdot b = b \cdot x$; *and* $5°$ *for every* $x_1^n \in Q$, $A(x_1^n) = x_1 \cdot \varphi(x_2) \cdot \ldots \cdot \varphi^{n-1}(x_n) \cdot b$.

1.5. Definition [15]: *We say that an algebra $(Q,\{\cdot,\varphi,b\})$ is a Hosszú–Gluskin algebra of order $n(n\geq 3)$ [briefly: nHG–algebra] iff $1^\circ - 4^\circ$ from 1.4 hold. In addition, we say that an nHG–algebra $(Q,\{\cdot,\varphi,b\}))$ is* **associated** *to the n–group (Q,A) iff 5° from 1.4 holds.*

1.6. Proposition [15]: *Let $n\geq 3$, let (Q,A) be an n–group, and* **e** *its $\{1,n\}$– neutral operation. Further on, let c_1^{n-2} be an arbitrary sequence over Q and let for every $x,y\in Q$*

$$B_{(c_1^{n-2})}(x,y)\overset{def}{=}A\left(x,c_1^{n-2},y\right),$$
$$\varphi_{(c_1^{n-2})}(x)\overset{def}{=}A\left(\mathbf{e}\left(c_1^{n-2}\right),x,c^{n-2}\right) \quad and$$
$$b_{(c_1^{n-2})}\overset{def}{=}A\left(\overline{\overset{n}{\mathbf{e}(c_1^{n-2})|}}\right).$$

Then, the following statements hold

(i) $(Q,\{B_{(c_1^{n-2})},\varphi_{(c_1^{n-2})},b_{(c_1^{n-2})})\}$ is an nHG–algebra associated to the n–group (Q,A); and

(ii) $C_A\overset{def}{=}\{(Q,\{B_{(c_1^{n-2})},\varphi_{(c_1^{n-2})},b_{(c_1^{n-2})}\})|c_1^{n-2}$ is a **sequence** *over $Q\}$ is the set of all nHG–algebras* **associated** *to the n–group (Q,A).*

1.7. Remark: Let (Q,F) be an m–groupoid and $m\in N$. Let, also, S_i be a subset of Q and $S_i\neq\emptyset$. Moreover, let

$$\mathbf{F}(S_1^m)\overset{def}{=}\bigcup_{(x_1^m)\in S_1\times\ldots\times S_m}F(x_1^m).$$

However, instead of **F**, usually, we write F.

1.8. Definition: *Let (Q,F) be an m–groupoid, $m\geq 1$ and let Q be equipped with a topology $\mathcal{O}$. Then, the m–ary operation F is continuous in $\mathcal{O}$ iff for every $x_1^m\in Q$ the following statement holds*

$$\left(\forall O_{F(x_1^m)}\in\mathcal{O}\right)(\exists O_{x_i}\in\mathcal{O})_1^m\, F(O_{x_1},\ldots,O_{x_m})\subseteq O_{(x_1^m)}.$$
$$[O_z\in\mathcal{O},z\in O_z,1.7.]$$

2. Main result

2.1. Definition: *Let (Q, A) be an $n-$group, $^{-1}$ its inversing operation [: [13, 16], 1.3], $n \geq 2$ and let Q be equipped with a topology $\mathcal{O}$. Then, we say that $(Q, A, \mathcal{O})$ is a topological $n-$group iff*

1° the $n-$ary operation a is continuous in $\mathcal{O}$ [:1.8], and

2° the $(n-1)-$ary operation $^{-1}$ is continuous in $\mathcal{O}$ [:1.8]. [2)] □

2.2. Remark: Topological $n-$groups have been defined in mutually different ways. Each one of these definitions was different from the description given in Definition 2.1. In addition, the definitions in [8] and [11] are related to each $n \geq 2$, while those in [9] and [10] are restricted (only) to each $n \geq 3$. Something in this connection will be said in section 3. □

2.3. Theorem: *Let (Q,A) be an $n-$group, $n \geq 3$ and let $(Q, \{\cdot, \varphi, b\})$ be an arbitrary $nHG-$algebra associated to the $n-$group (Q, A) [:[15],1.5]. Also, let Q be equipped with a topology $\mathcal{O}$. Then, $(Q, A, \mathcal{O})$ is a topological $n-$group iff the following statements hold*

(1) $(Q, \cdot, \mathcal{O})$ is a topolgical group [:e.g.[7]] and

(2) the unary operation φ is continuous in $\mathcal{O}$. [3)]

Proof. 1) Let $(Q, A, \mathcal{O})$ be a topological $n-$group [:2.1] and let $n \geq 3$. Also, let $\mathbf{e}$ and $^{-1}$, respectively, be an $\{1, n\}-$neutral operation and inversing operation of the $n-$group (Q, A) [:1.3,1.2]. In addition, let $(Q, \{\cdot, \varphi, b\})$ be an **arbitrary** $nHG-$ algebra associated to the $n-$group (Q, A) [:1.5]. Then, by Proposition 1.6, there is at least one sequence c_1^{n-1} over Q [$n \geq 3$] such that for every $x, y \in Q$ the following equalities hold

$$x \cdot y = A\left(x, c_1^{n-2}, y\right) \text{ and} \tag{3}$$

$$\varphi(x) = A\left(\mathbf{e}\left(c_1^{n-2}\right), x, c_1^{n-2}\right). \tag{4}$$

Let $^{-1}$ be an inversing operation of the group $(Q, \cdot)$. By (3), 1.2, 1.3

2)For $n = 2$ $(Q, A, \mathcal{O})$ is a topological group [:e.g. [7]].

3)The following proposition has been proved in [10]. If $(Q, A, \mathcal{O})$ is a topological $n-$group and $n \geq 3$, then there is an $nHG-$algebra $(Q, \{\cdot, \varphi, b\})$ [1.5] such that the following statements hold: 1. $(Q, \cdot, \mathcal{O})$ is a topological group, and 2. the unary operation φ is continuous in $\mathcal{O}$. See also footnote 9).

and 1.6, we conclude that for every $x \in Q$ the following equality holds

(5) $$x^{-1} = \left(c_1^{n-2}, x\right)^{-1};$$

$\mathbf{e}\left(c_1^{n-2}\right)$ is a neutral element of the group $(Q, \cdot)$.

In addition let (Q, F) be an $m-$groupoid and $m \in N$. Also, let S_{x_i} be a subset of Q and $x_i \in S_{x_i}$. Then, by the assumption $x_i \in S_{x_i}$, the following statements holds

(6) $$F\left(S_{x_i}, \ldots, S_{x_{i-1}}, \{x_i\}, S_{x_{i+1}}, \ldots, S_{x_m}\right) \subseteq F\left(S_{x_1}, \ldots, S_{x_{i-1}}, S_{x_i}, S_{x_{i+1}}, \ldots, S_{x_m}\right);\ x_i \in S_{x_i}, i \in \{1, \ldots, m\}.$$ [4)]

By the assumption $(Q, A, \mathcal{O})$ is a topological $n-$group, the following statements hold

(7) $$\left(\forall O_{A(x_1^n)} \in \mathcal{O}\right)\left(\exists O_{x_i} \in \mathcal{O}\right)_1^n A\left(O_{x_1}, \ldots, O_{x_n}\right) \subseteq O_{A(x_1^n)} \text{ and}$$

(8) $$\left(\forall O_{(x_1^{n-1})^{-1}} \in \mathcal{O}\right)\left(\exists O_{x_i} \in \mathcal{O}\right)_1^{n-1}\left(O_{x_1}, \ldots, O_{x_{n-1}}\right)^{-1} \subseteq O_{(x_1^{n-1})^{-1}} \quad [:1.7, 1.8].$$

Starting with the statements concerning (3) – (5) and by (6) – (8), we conclude that the following statements hold

$$(\forall O_{x\cdot y} \in \mathcal{O})(\exists O_x \in \mathcal{O})(\exists O_y \in \mathcal{O})\, O_x \cdot O_y \subseteq O_{x\cdot y},$$
$$\left(\forall O_{x^{-1}} \in \mathcal{O}\right)(\exists O_x \in \mathcal{O})\,(O_x)^{-1} \subseteq O_{x^{-1}} \text{ and}$$
$$\left(\forall O_{\varphi(x)} \in \mathcal{O}\right)(\exists O_x \in \mathcal{O})\,\varphi(O_x) \subseteq O_{\varphi(x)}.$$

2) Let $(Q, \{\cdot, \varphi, b\})$ be an $nHG-$algebra associated to the $n-$group (Q, A) [$n \geq 3$, 1.5]. Also, let Q be equipped with a topology $\mathcal{O}$. Further on, let the following statement hold

(1) $(Q, \cdot, \mathcal{O})$ is a topological group, and

(2) the unary operation φ is continuous in $\mathcal{O}$. Then $(Q, A, \mathcal{O})$ is a topological $n-$group.

Sketch of the proof.

[4)] See 1.7.

I_1 $(\forall O_{f(g(x))} \in \mathcal{O})(\exists O_{g(x)} \in \mathcal{O}) f(O_{g(x)}) \subseteq O_{f(g(x))}$ [:1.8, 1.7],
I_2 $(\forall O_{g(x)} \in \mathcal{O})(\exists O_x \in \mathcal{O}) g(O_x) \subseteq O_{g(x)}$ [:1.8, 1.7],
I_3 $T \subseteq S \Rightarrow f(T) \subseteq f(S)$ [: $T, S \in P(Q) \backslash \{\emptyset\}; f: Q \to Q$] and
I_4 $(\forall O_{f(g(x))} \in \mathcal{O})(\exists O_x \in \mathcal{O}) f(g(O_x)) \subseteq O_{f(g(x))}$ [:$I_1 - I_3$, 1.8].
II_1 $(\forall O_{f(x_1^m)\cdot g(x)} \in \mathcal{O})(\exists O_{f(x_1^m)} \in \mathcal{O})(\exists O_{g(x)} \in \mathcal{O}) O_{f(x_1^m)} \cdot O_{g(x)} \subseteq O_{f(x_1^m)\cdot g(x)}$ [:1.8],
II_2 $(\forall O_{f(x_1^m)} \in \mathcal{O})(\exists O_{x_i} \in \mathcal{O})_1^m f(O_{x_1}, \ldots, O_{x_m}) \subseteq O_{f(x_1^m)}$ [:1.8],
II_3 $(\forall O_{g(x)} \in \mathcal{O})(\exists O_x \in \mathcal{O}) g(O_x) \subseteq O_{g(x)}$ [:1.8] and
II_4 $(\forall O_{f(x_1^m)\cdot g(x)} \in \mathcal{O})(\exists O_{x_i} \in \mathcal{O})_1^m (\exists O_x \in \mathcal{O}) f(O_{x_1}, \ldots, O_{x_m}) \cdot g(O_x) \subseteq O_{f(x_1^m)\cdot g(x)}$ [: $II_1 - II_3$, 1.8]
III If $\psi(x) \stackrel{def}{=} a \cdot x$, then the operation ψ is continuous in $\mathcal{O}$ [:e.g. [7]].
IV_1 $A(x_1^n) = x_1 \cdot \varphi(x_2) \cdot \ldots \cdot \varphi^{n-2}(x_{n-1}) \cdot b \cdot x_n$ [:1.5],
IV_2 $\mathbf{e}(a_1^{n-2}) = (\varphi(a_1) \cdot \ldots \cdot \varphi^{n-2}(a_{n-2}) \cdot b)^{-1}$, where $^{-1}$ is the inversing operation of the group $(Q, \cdot)$ [: 1.2, 1.3, 1.5, IV_1],
IV_3 $(x_1^{n-2}, x)^{-1} = (\varphi(x_1) \cdot \ldots \cdot \varphi^{n-2}(x_{n-2}) \cdot b)^{-1} \cdot x^{-1} \cdot (\varphi(x_1) \cdot \ldots \cdot \varphi^{n-2}(x_{n-2}) \cdot b)^{-1}$ [:1.2, 1.3, 1.5, IV_2]. □

3. Three propositions more

3.1. Proposition: *Let (Q, A) be an $n-$group, $^{-1}$ its inversing operation [:[13,16], 1.3], $n \geq 2$, let A^{-1} be an $n-$ary operation in Q such that for every sequence x_1^{n-2} over Q and for all $x, y, z \in Q$ the following equivalence holds*

$$(\text{r}) \qquad A^{-1}\left(x, x_1^{n-2}, y\right) = z \Leftrightarrow A\left(x, x_1^{n-2}, z\right) = y,$$

and let ^{-1}A be an $n-$ary operation in Q such that for every sequence x_1^{n-2} over Q and for all $x, y, z \in Q$ the following equivalence holds

$$(\text{l}) \qquad {}^{-1}A\left(x, x_1^{n-2}, y\right) = z \Leftrightarrow A\left(z, x_1^{n-2}, y\right) = x.$$

Also, let Q be equipped with a topology $\mathcal{O}$. Further on, let the $n-$ary operation A be continuous in $\mathcal{O}$. Then the following statements are equivalent:
(i) the $(n-1)-$ary operation $^{-1}$ is continuous in $\mathcal{O}$; and

(ii) the $n-$ary operation A^{-1} is continuous in $\mathcal{O}$ and the $n-$ary operation ^{-1}A is continuous in $\mathcal{O}$.[5)]

Sketch of the proof.

I_1 $A^{-1}\left(x, x_1^{n-2}, y\right) = z \Leftrightarrow A\left(x, x_1^{n-2}, z\right) = y \Leftrightarrow$

$$A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, A\left(x, x_1^{n-2}, z\right)\right) = A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, y\right) \Leftrightarrow$$

$$A\left(A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, x\right), x_1^{n-2}, z\right) = A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, y\right) \Leftrightarrow$$

$$A\left(\mathbf{e}\left(x_1^{n-2}\right), x_1^{n-2}, z\right) = A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, y\right) \Leftrightarrow$$

$$z = A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, y\right) \text{ [:(}r\text{), 1.1, 1.2, 1.3].}$$

I_2 $A^{-1}\left(x, x_1^{n-2}, y\right) = A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, y\right)$ [:I_1].

I_3 $^{-1}A\left(x, x_1^{n-2}, y\right) = A\left(x, x_1^{n-2}, \left(x_1^{n-2}, y\right)^{-1}\right)$ [:(l), 1.1, 1.2, 1.3].

I_4 $A\left(\left(x_1^{n-2}, x\right)^{-1}, x_1^{n-2}, y\right) = A^{-1}\left(x, x_1^{n-2}, y\right) \Leftrightarrow$

$$^{-1}A\left(A^{-1}\left(x, x_1^{n-2}, y\right), x_1^{n-2}, y\right) = \left(x_1^{n-2}, x\right)^{-1} \text{ [:}I_2\text{, (}l\text{)].}$$

$$[A^{-1}\left(x, x_1^{n-2}, {}^{-1}A\left(x, x_1^{n-2}, y\right)\right) = \left(x_1^{n-2}, y\right)^{-1} \text{ [:}I_3\text{, (}r\text{)]].}$$

I_5 $\left(x_1^{n-2}, x\right)^{-1} = {}^{-1}A\left(A^{-1}\left(x, x_1^{n-2}, x\right), x_1^{n-2}, x\right)$ [:$I_4, y = x$].

II_1 $\left(\forall O_{F(f(x_1^m), y_1^t, g(z_1^s))} \in \mathcal{O}\right)\left(\exists O_{f(x_1^m)} \in \mathcal{O}\right)\left(\exists O_{y_i} \in \mathcal{O}\right)_1^t\left(\exists O_{g(z_1^s)} \in \mathcal{O}\right)$
$F\left(O_{f(x_1^m)}, \overline{O_{y_i}}|_{i=1}^t, O_{g(z_1^s)}\right) \subseteq O_{F(f(x_1^m), y_1^t, g(z_1^s))}$ [:1.8].

II_2 $\left(\forall O_{f(x_1^m)} \in \mathcal{O}\right)\left(\exists O_{x_i} \in \mathcal{O}\right)_1^m f\left(O_{x_1}, \ldots, O_{x_m}\right) \subseteq O_{f(x_1^m)}$ [:1.8]

II_3 $\left(\forall O_{g(z_1^s)} \in \mathcal{O}\right)\left(\exists O_{z_i} \in \mathcal{O}\right)_1^s g\left(O_{z_1}, \ldots, O_{z_s}\right) \subseteq O_{g(z_1^s)}$ [:1.8]

II_4 $\left(\forall O_{F(f(x_1^m), y_1^t, g(z_1^s))} \in \mathcal{O}\right)\left(\exists O_{x_i} \in \mathcal{O}\right)_1^m\left(\exists O_{y_i} \in \mathcal{O}\right)_1^t\left(\exists O_{z_i} \in \mathcal{O}\right)_1^s$

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5) In [8], $(Q, A, \mathcal{O})$ is a topological $n-$group iff following statements hold: (a) the $n-$ary operation A is continuous $\mathcal{O}$, (b) the $n-$ary operation A^{-1} is continuous in $\mathcal{O}$, and (c) the $n-$ary operation ^{-1}A is continuous in $\mathcal{O}$.

$F(f(O_{x_1},...,O_{x_m}),O_{y_1},...,O_{y_t},g(O_{z_1},...,O_{z_s})) \subseteq O_{F(f(x_1^m),y_1^t,g(z_1^s))}$ *[:*$II_1 - II_3$, 1.8*].* □

3.2. Proposition: *Let* (Q,A) *be an* $n-$*group,* $^{-1}$ *its inversing operation [:*[13,16], *1.3], $n \geq 2$ and let* $^{(-2)}$ *be an unary operation in* Q *such that for every* $x \in Q$ *the following equality holds*

$$\text{(u)} \qquad \overset{2}{A}\left(x^{(-2)}, \overset{2n-2}{x}\right) = x,$$

where $\overset{2}{A}(x_1^{2n-1}) \overset{def}{=} A(A(x_1^n), x_{n+1}^{2n-1})$. *Also, let* Q *be equipped with a topology* $\mathcal{O}$. *Further on, let the* $n-$*ary operation* A *be continuous in* $\mathcal{O}$. *Then the following statements are equivalent:*

$(\bar{i})$ *the* $(n-1)-$*ary operation* $^{-1}$ *is continuous in* $\mathcal{O}$; *and*

$(\overline{ii})$ *the unary operation* $^{(-2)}$ *is continuous in* $\mathcal{O}$.[6)]

Sketch of the proof.

$(\bar{i}) \Rightarrow (\overline{ii})$:

I_1 $\overset{2}{A}\left(x^{(-2)}, \overset{2n-3}{x}, x\right) = x \Leftrightarrow$

$A\left(x^{(-2)}, \overset{n-2}{x}, A\left(x, \overset{n-2}{x}, x\right)\right) = A\left(\left(\overset{n-2}{x}, x\right)^{-1}, \overset{n-2}{x}, A\left(x, \overset{n-2}{x}, x\right)\right) \Leftrightarrow$

$x^{(-2)} = \left(\overset{n-2}{x}, x\right)^{-1}$ *[:*(u),1.2,1.3,1.1*].*

I_2 $\left(\forall O_{(x_1^{n-1})^{-1}} \in \mathcal{O}\right)\left(\exists O_{x_i} \in \mathcal{O}\right)_1^{n-1}\left(O_{x_1},...,O_{x_{n-1}}\right)^{-1} \subseteq O_{(x_1^{n-1})^{-1}}$ *[:*$(\bar{i})$, 1.8*].*

I_3 $(\forall O_{x^{(-2)}} \in \mathcal{O})(\exists O_x \in \mathcal{O})(O_x)^{(-2)} \subseteq O_{x^{(-2)}}$ *[:*I_1, I_2, 1.8*].*

$(\overline{ii}) \Rightarrow (\bar{i})$:

II_1 $A\left(\overset{n-3}{A}\left(\mathbf{e}\left(\overset{n-2}{a}_{n-2}\right), \overset{n-3}{a}_{n-2}, ..., \mathbf{e}\left(\overset{n-3}{a_2}\right), \overset{n-3}{a_2}, \mathbf{e}\left(\overset{n-2}{a_1}\right), \overset{n-3}{a_1}\right), a_1^{n-2}, x\right) =$

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$A\left(\mathbf{e}\left(\overset{n-2}{a}_{n-2}\right), \overset{n-3}{a}_{n-2}, a_{n-2}, x\right) = x$ *[:* $\overset{1}{A} \overset{def}{=} A$, $\overset{k+1}{A}\left(x_1^{(k+1)(n-1)+1}\right) \overset{def}{=}$

6)In [11], $(Q,A,\mathcal{O})$ is a topological $n-$group iff the following statement hold: $(\bar{a})$ the $n-$ary operation A is continuous in $\mathcal{O}$, and $(\bar{b})$ the unary operation $^{(-2)}$ is continuous in $\mathcal{O}$.

$A\left(\overset{k}{A}\left(x_1^{k(n-1)+1}\right), x_{k(n-1)+2}^{(k+1)(n-1)+1}\right)$, for $k = 0$ $\overset{k}{A}\left(x_1^{k(n-1)+1}\right) \overset{def}{=} x_1$; 1.1, 1.3 /.

II_2 $A\left(\overset{n-3}{A}\left(\mathbf{e}\left(\overset{n-2}{a}_{n-2}\right), \overset{n-3}{a}_{n-2}, \ldots, \mathbf{e}\left(\overset{n-2}{a_1}\right), \overset{n-3}{a_1}\right), a_1^{n-2}, x\right) =$ $A\left(\mathbf{e}\left(a_1^{n-2}\right), a_1^{n-2}, x\right)$ /: II_1, 1.3/.

II_3 Let (Q, A) be an n–group, $\mathbf{e}$ its $\{1, n\}$– neutral operation, and let $n \geq 3$. Then for every sequence a_1^{n-2} over Q the following equality holds

$$\mathbf{e}\left(a_1^{n-2}\right) = \overset{n-3}{A}\left(\mathbf{e}\left(\overset{n-2}{a}_{n-2}\right), \overset{n-3}{a}_{n-2}, \ldots, \mathbf{e}\left(\overset{n-2}{a_1}\right), \overset{n-3}{a_1}\right) \text{ [: } II_2, 1.1 \text{].}^{7)}$$

II_4 $(a_1^{n-2}, a)^{-1} \overset{def}{=} \mathsf{E}(a_1^{n-2}, a, a_1^{n-2})$, where E is an $\{1, 2n-1\}$–neutral operation of the $(2n-1)$–group $(Q, \overset{2}{A})$ /:1.3/.

II_5 $(x_1^{n-2}, x)^{-1} = \overset{2(n-2)}{A}((\overset{n-1}{x}_{n-2})^{-1}, \overset{2n-4}{x}_{n-2}, \ldots,$ $(\overset{n-1}{x_1})^{-1}, \overset{2n-4}{x_1}, (\overset{n-1}{x})^{-1}, \overset{2n-4}{x}, (\overset{n-1}{x}_{n-2})^{-1}, \ldots, \overset{2n-4}{x_1})$ /:II_4, II_3/.

3.3. Proposition: *Let (Q, A) be an n–group, $\mathbf{e}$ its $\{1, n\}$–neutral operation, $^{-1}$ its inversing operation, $n \geq 3$, and let $^{-}$ be an unary operation in Q such that for every $x \in Q$ the following equality holds*

$$\text{(v)} \qquad \bar{x} = \mathbf{e}\left(\overset{n-2}{x}\right) \; ^{8)}$$

Also, let Q be equipped with a topology $\mathcal{O}$. Further on, let the n–ary operation A is continuous in $\mathcal{O}$. Then the following statements are equivalent:

($\bar{\bar{i}}$) *the $(n-1)$–ary operation $^{-1}$ is continuous in $\mathcal{O}$;*

($\overline{\overline{ii}}$) *the $(n-2)$–ary operation $\mathbf{e}$ is continuous in $\mathcal{O}$; and*

($\overline{\overline{iii}}$) *the unary operation $^{-}$ is continuous in $\mathcal{O}$.*[9].

[7]The assertion II_3 is actually Theorem 2.1 from [17].

[8]Actually, the unary operation **skew element** /:[1]; see e.g. [5,6]/ is in question.

[9]In [9] and [10], $(Q, A, \mathcal{O})$ is a topological n–group, $n \geq 3$, iff the following statements hold: ($\bar{\bar{a}}$) the n–ary operation A is continuous in $\mathcal{O}$, and ($\bar{\bar{b}}$) the unary operation $^{-}$ is continuous in $\mathcal{O}$.

Sketch of the proof.

$(\overline{\overline{i}}) \Leftrightarrow (\overline{\overline{ii}})$:

I_1 $\mathbf{e}(a_1^{n-2}) = A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a)$ /:1.2,1.3/.

I_2 $\mathbf{e}(a_1^{n-2}) = A((a_1^{n-2}, a_1)^{-1}, a_1^{n-2}, a_1)$ /:$I_1, a = a_1$; for "$\Rightarrow$" /.

I_3 $A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a) = \mathbf{e}(a_1^{n-2}) \Leftrightarrow$
$A(A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a), \overset{n-3}{a}, \mathbf{e}(\overset{n-2}{a}), \mathbf{e}(a_1^{n-2})) =$
$A(\mathbf{e}(a_1^{n-2}), \overset{n-3}{a}, \mathbf{e}(\overset{n-2}{a}), \mathbf{e}(a_1^{n-2})) \Leftrightarrow$
$A((a_1^{n-2}, a)^{-1}, a_1^{n-3}, A(a_{n-2}, a, \overset{n-3}{a}, \mathbf{e}(\overset{n-2}{a})), \mathbf{e}(a_1^{n-2})) =$
$A(\mathbf{e}(a_1^{n-2}), \overset{n-3}{a}, \mathbf{e}(\overset{n-2}{a}), \mathbf{e}(a_1^{n-2})) \Leftrightarrow$
$A((a_1^{n-2}, a)^{-1}, a_1^{n-3}, a_{n-2}, \mathbf{e}(a_1^{n-2})) =$
$A(\mathbf{e}(a_1^{n-2}), \overset{n-3}{a}, \mathbf{e}(\overset{n-2}{a}), \mathbf{e}(a_1^{n-2})) \Leftrightarrow$
$(a_1^{n-2}, a)^{-1} = A(\mathbf{e}(a_1^{n-2}), \overset{n-3}{a}, \mathbf{e}(\overset{n-2}{a}), \mathbf{e}(a_1^{n-2}))$ /: $I_1, n \geq 3$, 1.1, 1.3; for "$\Leftarrow$" /.

$(\overline{\overline{ii}}) \Leftrightarrow (\overline{\overline{iii}})$:

II_1 $\overline{x} = \mathbf{e}(\overset{n-2}{x})$ /:(v); for "$\Rightarrow$" /.

II_2 $\mathbf{e}(x_1^{n-2}) = \overset{n-3}{A}(\overline{x}_{n-2}, \overset{n-3}{x}_{n-2}, \ldots, \overline{x}_1, \overset{n-3}{x_1})$ /: the sketch of the proof of 3.2–II_3, (v); for "$\Leftarrow$"/. □

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Received January 20, 1998.