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Intuitionistic Fuzzy Strongly Preopen (Preclosed) Mappings

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Abstract. Intuitionistic fuzzy strongly preopen (preclosed) mappings between intuitionistic fuzzy topological spaces are introduced. Some of their properties are studied.

1. Introduction

The concept of fuzzy set was introduced by Zadeh in his classic paper [10]. Using the concept of fuzzy sets Chang [2] introduced the fuzzy topological spaces. Since Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] defined the intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. One of directions is related to the intuitionistic fuzzy generalized set introduced by Coker [4]. Continuing this work, Jeon [6] get some deeper results about the intuitionistic fuzzy generalized sets, intuitionistic α-continuity and the intuitionistic fuzzy precontinuity.

In this paper, as an extension of authors concept presented in [8], we will introduce intuitionistic fuzzy strongly preopen (preclosed) mappings and we will study some of their properties. We will establish their relationships with other weaker forms of intuitionistic fuzzy continuous mappings.

2. Preliminaries

We introduce some basic notions and results that are used in the sequel.

Definition 2.1 ([1])**.** Let X be a nonempty fixed set and I the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{\langle X, \mu_A(X), \nu_A(X)\rangle \mid X \in X\},$$

where the mapping $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(X)$) and the degree of nonmembership (namely $\nu_A(X)$) for each

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element $X \in X$ to the set A, respectively, and $0 \leq \mu_A(X) + \nu_A(X) \leq 1$ for each $X \in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{\langle X, \mu_A(X), 1 - \mu_A(X)\rangle \mid X \in X\}.$$

Definition 2.2 ([1])**.** Let A and B be IFSs of the form

$$A = \{\langle X, \mu_A(X), \nu_A(X)\rangle \mid X \in X\}$$

and

$$B = \{\langle X, \mu_B(X), \nu_B(X)\rangle \mid X \in X\}.$$

Then

(i) $A \subseteq B$ if and only if $\mu_A(X) \leq \mu_B(X)$ and $\nu_A(X) \geq \nu_B(X)$;
(ii) $\overline{A} = \{\langle X, \nu_A(X), \mu_A(X)\rangle \mid X \in X\}$;
(iii) $A \cap B = \{\langle X, \mu_A(X) \wedge \mu_B(X), \nu_A(X) \vee \nu_B(X)\rangle \mid X \in X\}$;
(iv) $A \cup B = \{\langle X, \mu_A(X) \vee \mu_B(X), \nu_A(X) \wedge \nu_B(X)\rangle \mid X \in X\}$.

We will use the notation $A = \langle X, \mu_A, \nu_A\rangle$ instead of $A = \{\langle X, \mu_A(X), \nu_A(X)\rangle \mid X \in X\}$. A constant fuzzy set α taking value $\alpha \in [0,1]$ will be denote by $\underline{\alpha}$.

The IFSs $\underset{\sim}{0}$ and $\underset{\sim}{1}$ are defined by $\underset{\sim}{0} = \{\langle X, \underline{0}, \underline{1}\rangle \mid X \in X\}$ and $\underset{\sim}{1} = \{\langle X, \underline{1}, \underline{0}\rangle \mid X \in X\}$.

Let f be a mapping from an ordinary set X into an ordinary set Y. If $B = \{\langle Y, \mu_B(Y), \nu_B(Y)\rangle \mid Y \in Y\}$ is an IFS in Y, then the inverse image of B under f is IFS defined by

$$F^{-1}(B) = \{\langle(X, F^{-1}(\mu_B)(X), F^{-1}(\nu_B)(X)\rangle \mid X \in X\}.$$

The image of IFS $A = \{\langle Y, \mu_A(Y), \nu_A(Y)\rangle \mid Y \in Y\}$ under f is IFS defined by

$$F(A) = \{\langle Y, F(\mu_A)(Y), F(\nu_A)(Y)\rangle \mid Y \in Y\},$$

where

$$F(\mu_A)(y) = \begin{cases} \sup\limits_{x \in F^{-1}(y)} \mu_A(x), & f^{-1}(y) \neq 0, \\ 0, & \text{otherwise} \end{cases}$$

and

$$F(\nu_A)(y) = \begin{cases} \inf\limits_{x \in F^{-1}(y)} \nu_A(x), & f^{-1}(y) \neq 0, \\ 1, & \text{otherwise} \end{cases}$$

for each $y \in Y$.

Definition 2.3 ([3])**.** An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(T_1) $\underset{\sim}{0}, \underset{\sim}{1} \in \tau$;
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(T_3) $\cup G_I \in \tau$ for any arbitrary family $\{G_I \mid I \in J\} \subseteq \tau$.

In this paper by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS in τ is called intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4 ([3])**.** Let be an IFS in IFTS X. Then

$\operatorname{int} A = \cup\{G \mid G$ is an IFOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy interior of A;

$\operatorname{cl} A = \cap\{G \mid G$ is an IFCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy closure of A.

Definition 2.5 ([4])**.** An IFS A in an IFTS X is called

(i) an intuitionistic fuzzy α-open set (IFαOS) if $A \subseteq \operatorname{int}(\operatorname{cl}(\operatorname{int} A))$;
(ii) an intuitionistic fuzzy semiopen set (IFSOS) if $A \subseteq \operatorname{cl}(\operatorname{int} A)$;
(iii) an intuitionistic fuzzy preopen set (IFPOS) if $A \subseteq \operatorname{int}(\operatorname{cl} A)$;

Definition 2.6 ([9])**.** Let be an IFS in IFTS X. Then

$\operatorname{pint} A = \cup\{G \mid G$ is an IFPOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy preinterior of A;

$\operatorname{pcl} A = \cap\{G \mid G$ is and IFPCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy preclosure of A.

Definition 2.7 ([9])**.** An IFS A in an IFTS X is called an intuitionistic fuzzy strongly preopen set (IFSPOS) if $A \subseteq \operatorname{int}(\operatorname{pcl} A)$.

An IFS A is called an intuitionistic fuzzy α-closed set, an intuitionistic fuzzy semiclosed set, an intuitionistic fuzzy preclosed set and an intuitionistic fuzzy strongly preclosed set, respectively (IFαCS, IFSCS, IFPCS and IFSPCS) if the complement of A is an IFαOS, IFSOS, IFPOS and IFSPOS, respectively.

Definition 2.8 ([9])**.** Let be an IFS in IFTS X. Then

$\operatorname{spint} A = \cup\{G \mid G$ is an IFPOS in X and $G \subseteq A\}$ is called an intuitionistic fuzzy strongly preinterior of A;

$\operatorname{spcl} A = \cap\{G \mid G$ is an IFPCS in X and $G \supseteq A\}$ is called an intuitionistic fuzzy strongly preclosure of A.

Theorem 2.1 ([3, 9])**.** *Let A be an IFS of IFTS X. Then*

(i) $\operatorname{cl} \overline{A} = \overline{\operatorname{int} A}$;
(ii) $\operatorname{int} \overline{A} = \overline{\operatorname{cl} A}$;
(iii) $\operatorname{pcl} \overline{A} = \overline{\operatorname{pint} A}$;
(iv) $\operatorname{pint} \overline{A} = \overline{\operatorname{pcl} A}$;
(v) $\operatorname{spcl} \overline{A} = \overline{\operatorname{spint} A}$;
(vi) $\operatorname{spint} \overline{A} = \overline{\operatorname{spcl} A}$.

Definition 2.9 ([6, 9])**.** Let f be a mapping from an IFTS X into an IFTS Y. The mapping f is called:

(i) an intuitionistic fuzzy continuous if $f^{-1}(B)$ is an IFOS in X, for each IFOS B in Y.
(ii) an intuitionistic fuzzy α-continuous if $f^{-1}(B)$ is an IFαOS in X, for each IFOS B in Y;

(iii) an intuitionistic fuzzy precontinuous if $f^{-1}(B)$ is an IFPOS in X, for each IFOS B in Y.
(iv) an intuitionistic fuzzy strong precontinuous if $f^{-1}(B)$ is an IFSPOS in X, for each IFOS B in Y.

Definition 2.10 ([6, 9])**.** Let f be a mapping from an IFTS X into IFTS Y. The mapping f is called:

(i) an intuitionistic fuzzy open (closed) if $f(A)$ is IFOS of Y, for each IFOS (IFCS) A of X;
(ii) an intuitionistic fuzzy α-open (α-closed) $f(A)$ is IFαOS (IFαCS) of Y, for each IFOS (IFCS) A of X;
(iii) an intuitionistic fuzzy preopen (preclosed) if $f(A)$ is IFPOS (IFPCS) of Y, for each IFOS (IFCS) A of X.

3. INUITIONISTIC FUZZY STRONGY PREOPEN (PRECLOSED) MAPPINGS

Definition 3.1. A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called intuitionistic fuzzy strongly preopen (preclosed) if $f(A)$ is IFSPOS (IFSPCS) of Y, for each IFOS (IFCS) A of X.

Remark 3.1. The following diagram of implications is true.

$$\begin{array}{ccc}
\boxed{\text{intuitionistic fuzzy open (closed)}} & \Rightarrow & \boxed{\text{intuitionistic fuzzy } \alpha\text{-open (closed)}} \\
\Downarrow & & \Downarrow \\
& \boxed{\text{intuitionistic fuzzy strongly preopen (preclosed)}} & \\
& \Downarrow & \\
& \boxed{\text{intuitionistic fuzzy preopen (preclosed)}} &
\end{array}$$

With the following example we can shows that the reverse may not be true.

Example 3.1. Let $X = \{A, B, C\}$ and A, B and C are IFSs defined by

$$A = \left\langle X, \left(\frac{a}{0,3}, \frac{b}{0,2}, \frac{c}{0,7}\right), \left(\frac{a}{0,6}, \frac{b}{0,8}, \frac{c}{0,3}\right)\right\rangle$$
$$B = \left\langle X, \left(\frac{a}{0,8}, \frac{b}{0,8}, \frac{c}{0,4}\right), \left(\frac{a}{0,1}, \frac{b}{0,2}, \frac{c}{0,6}\right)\right\rangle$$
$$C = \left\langle X, \left(\frac{a}{0,8}, \frac{b}{0,7}, \frac{c}{0,6}\right), \left(\frac{a}{0,1}, \frac{b}{0,3}, \frac{c}{0,4}\right)\right\rangle.$$

Let $\tau_1 = \{\underset{\sim}{0}, C, \underset{\sim}{1}\}$, $\tau_2 = \{\underset{\sim}{0}, A, B, A \cap B, A \cup B, \underset{\sim}{1}\}$, $\tau_3 = \{\underset{\sim}{0}, A, \underset{\sim}{1}\}$ and $\tau_4 = \{\underset{\sim}{0}, B, \underset{\sim}{1}\}$. Then the mapping $f = \mathrm{id} : (X, \tau_1) \to (X, \tau_2)$ is intuitionistic fuzzy strongly preopen (preclosed) but it is not intuitionistic fuzzy α-open (α-closed). Also, F is not intuitionistic fuzzy open (closed). Similarly, the mapping $F = \mathrm{id} : (X, \tau_3) \to (X, \tau_4)$ is intuitionistic fuzzy preopen (preclosed), but it is not intuitionistic fuzzy strongly preopen (preclosed).

Theorem 3.1. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy strongly preopen (preclosed) mappimg if and only if it is an intuitionistic fuzzy strongly preclosed (preopen) mapping.*

Proof. It can be proved by using the complement. □

Theorem 3.2. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy strongly preopen (preclosed) mapping if and only if f^{-1} is intuitionistic fuzzy strongly precontinuous mapping.*

Proof. It follows from the relation $(f^{-1})^{-1}(A) = f(A)$, for each IFOS (IFCS) set A of X. □

Theorem 3.3. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy strongly preopen mapping if and only if*

$$f(\operatorname{int} A) \subseteq \operatorname{spint} f(A),$$

for each IFSCS A of X.

Proof. Let f be an intuitionistic fuzzy strongly preopen mapping and let A be any IFSCS of X. Then

$$f(\operatorname{int} A) = \operatorname{spint} f(\operatorname{int} A) \subseteq \operatorname{spint} f(A).$$

Conversely, let A be any IFOS of X. According to the assumption we have

$$f(A) = f(\operatorname{int} A) \subseteq \operatorname{spint} f(A).$$

Thus $f(A)$ is an IFSPOS of X, so f is an intuitionistic fuzzy strongly preopen mapping. □

Theorem 3.4. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy strongly preclosed mapping if and only if* $\operatorname{spcl} f(A) \subseteq F(\operatorname{cl} A)$, *for each IFSOS A of X.*

Proof. It can be proved in a similar manner as Theorem 3.3. □

Theorem 3.5. *Let $f : X \to Y$ be a bijective mapping from an IFTS X into an IFTS Y. Then the following statements are equivalent:*

(i) *f is an intuitionistic fuzzy strongly preopen (preclosed) mapping;*
(ii) $f(\operatorname{int} A) \subseteq \operatorname{spint} f(A)$, *for each IFSCS A of X;*
(iii) $\operatorname{spcl} f(A) \subseteq f(\operatorname{cl} A)$, *for each IFSOS A of X.*

Proof. It follows from the Theorem 3.1, Theorem 3.3 and Theorem 3.4. □

Theorem 3.6. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statements hold:*

(1) *f is an intuitionistic fuzzy strongly preopen mapping if and only if* $f(\operatorname{int} A) \subseteq \operatorname{int}(\operatorname{pcl} f(A))$, *for each IFSCS A of X.*
(2) *f is an intuitionistic fuzzy strongly preclosed mapping if and only if* $\operatorname{cl}(\operatorname{pint} f(A) \subseteq f(\operatorname{cl} A)$, *for each IFS A of X.*

Proof. We will prove the statements (1) only. Let f be an intuitionistic fuzzy strongly preopen mapping. Then, $f(\operatorname{int} A)$ is an IFSPOS of Y. Therefore

$$f(\operatorname{int} A) \subseteq \operatorname{int}(\operatorname{pcl} f(\operatorname{int} A)) \subseteq \operatorname{int}(\operatorname{pcl} f(A)).$$

Conversely, let A be any IFOS of X. From $f(A) = f(\operatorname{int} A) \subseteq \operatorname{int}(\operatorname{pcl} f(A))$, it follows that $f(A)$ is an IFSPOS, so f is an intuitionistic fuzzy strongly preopen mapping. □

Theorem 3.7. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is intuitionistic fuzzy strongly preopen if and only if for each fuzzy set B of Y and each IFCS A of X, when $f^{-1}(B) \subseteq A$, there exists an IFSPCS C of Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. Let B be any IFS of Y and let A be an IFCS of X such that $f^{-1}(B) \subseteq A$. Then $\overline{A} \subseteq f^{-1}(\overline{B})$, so $f(\overline{A}) \subseteq ff^{-1}(\overline{B}) \subseteq \overline{B}$. Since $\overline{A}$ is an IFOS, $f(\overline{A})$ is an IFSPOS, so $f(\overline{A}) \subseteq \operatorname{spint} \overline{B}$. Hence $\overline{A} \subseteq f^{-1}f(\overline{A}) \subseteq f^{-1}(\operatorname{spint} \overline{B})$. Therefore $A \supseteq \overline{f^{-1}f(\overline{B})} = f^{-1}(\operatorname{spcl} B)$. The result follows for $C = \operatorname{spcl} B$.

Conversely, let U be any IFOS of X. We will show that $f(U)$ is an IFSPOS of Y. From $U \subseteq f^{-1}f(U)$ follows that $\overline{U} \supseteq \overline{f^{-1}f(U)} \supseteq f^{-1}\overline{f(U)}$ where $\overline{U}$ is an IFCS of X. Hence there is an IFSPCS B of Y such that $B \supseteq \overline{f(U)}$ and $f^{-1}(B) \subseteq \overline{U}$. From $B \supseteq \overline{f(U)}$ follows that $B \supseteq \operatorname{spcl} \overline{F(U)}$, so $\overline{B} \subseteq \overline{\operatorname{spcl} \overline{f(U)}} \subseteq \operatorname{spint} f(U)$.

From $f^{-1}(B) \subseteq \overline{U}$ we have $B \supseteq f^{-1}(\overline{B}) \supseteq U$, so $\overline{B} \supseteq ff^{-1}(\overline{B}) \supseteq f(U)$. Hence $f(U) = \operatorname{spint} f(U)$.

Thus $f(U)$ is an IFSPOS, so f is a intuitionistic fuzzy strongly preopen mapping. □

Theorem 3.8. *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then f is an intuitionistic fuzzy strongly preclosed mapping if and only if for each IFS set B of Y and each IFOS A of X, when $f^{-1}(B) \subseteq A$, there exists an IFSPOS C of Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. It can be proved in a similar manner as Theorem 3.7. □

Theorem 3.9. *Let $f : X \to Y$ and $g : Y \to Z$ are mappings, where X, Y and Z are IFTS. If g is intuitionistic fuzzy strongly preopen (preclosed) and f is intuitionistic fuzzy open (closed), then gf is intuitionistic fuzzy strongly preopen (preclosed).*

Proof. For any IFOS (IFCS) A of X we have $(gf)(A) = g(f(A))$. Since f is intuitionistic fuzzy open (closed) and g is intuitionistic fuzzy strongly preopen (preclosed), we obtain that (gf)(A) is an IFSPOS (IFSPCS) of Z. □

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